

QUESTIONS & SOLUTIONS OF AIEEE 2009

Date : 26-04-2009

Duration : 3 Hours

Max. Marks : 432

Important Instructions:

1. The Test Booklet consists of **90** questions. The maximum marks are **432**.
2. There are three parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

Part A — CHEMISTRY (144 marks)
Question No. **1 to 24** consist **FOUR (4)** marks each and Question No. **25 to 30** consist **EIGHT (8)** marks each for each correct response.

Part B — MATHEMATICS (144 marks)
Question No. **31 to 32** and **39 to 60** consist **FOUR (4)** marks each and Question No. **33 to 38** consist **EIGHT (8)** marks each for each correct response.

Part C — PHYSICS (144 marks)
Question No. **61 to 84** consist **FOUR (4)** marks each and Question No. **85 to 90** consist **EIGHT (8)** marks each for each correct response.
3. Candidates will be awarded marks as stated above in instructions No. 2 for correct response of each question. $\frac{1}{4}$ (**one fourth**) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

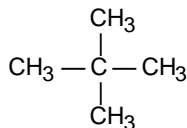
Where you will be in resonance with IIT-JEE

PART-A (CHEMISTRY)

1. The IUPAC name of neopentane is :
(1) 2, 2-dimethylpropane (2) 2-methylpropane
(3) 2, 2-dimethylbutane (4) 2-methylbutane

Ans. 1

Sol.



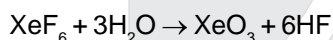
2, 2-dimethylpropane

2. Which one of the following reactions of Xenon compounds is not feasible ?
(1) $3\text{XeF}_4 + 6\text{H}_2\text{O} \rightarrow 2\text{Xe} + \text{XeO}_3 + 12\text{HF} + 1.5\text{O}_2$ (2) $2\text{XeF}_2 + 2\text{H}_2\text{O} \rightarrow 2\text{Xe} + 4\text{HF} + \text{O}_2$
(3) $\text{XeF}_6 + \text{RbF} \rightarrow \text{Rb}[\text{XeF}_7]$ (4) $\text{XeO}_3 + 6\text{HF} \rightarrow \text{XeF}_6 + 3\text{H}_2\text{O}$

Ans. 4

Sol.

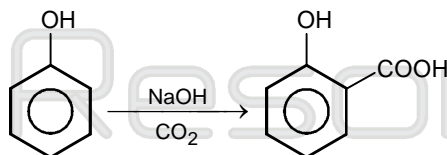
This reaction is not feasible because XeF_6 formed will get hydrolysed according to the following reaction,



3. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is :
(1) salicylaldehyde (2) salicylic acid (3) phthalic acid (4) benzoic acid

Ans. 2

Sol.



Salicylic acid

4. Which of the following statements is incorrect regarding physisorptions ?
(1) More easily liquefiable gases are adsorbed readily.
(2) Under high pressure it results into multi molecular layer on adsorbent surface.
(3) Enthalpy of adsorption ($\Delta H_{\text{adsorption}}$) is low and positive.
(4) It occurs because of van der Waal's forces.

Ans. 3

Sol.

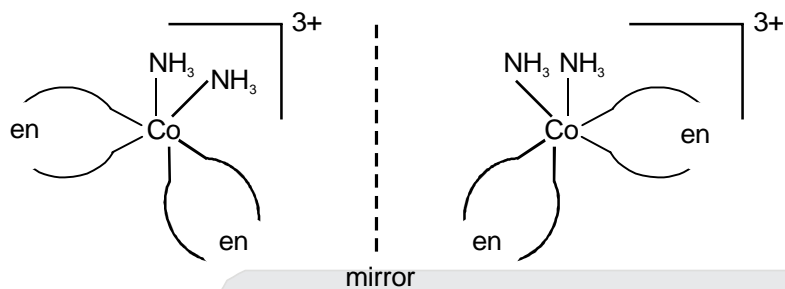
Since adsorption is exothermic process so ΔH of adsorption is always negative.

5. Which of the following has an optical isomer ?
 (1) $[\text{Co}(\text{en})(\text{NH}_3)_2]^{2+}$ (2) $[\text{Co}(\text{H}_2\text{O})_4(\text{en})]^{3+}$ (3) $[\text{Co}(\text{en})_2(\text{NH}_3)_2]^{3+}$ (4) $[\text{Co}(\text{NH}_3)_3\text{Cl}]^+$

Ans. 3

Sol.

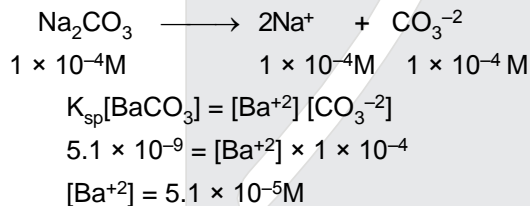
cis-form of $[\text{Co}(\text{en})_2(\text{NH}_3)_2]^{3+}$ has optical isomers.



6. Solid $\text{Ba}(\text{NO}_3)_2$ is gradually dissolved in $1.0 \times 10^{-4} \text{ M}$ Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form ? (K_{sp} for $\text{BaCO}_3 = 5.1 \times 10^{-9}$) :
 (1) $5.1 \times 10^{-5} \text{ M}$ (2) $8.1 \times 10^{-8} \text{ M}$ (3) $8.1 \times 10^{-7} \text{ M}$ (4) $4.1 \times 10^{-5} \text{ M}$

Ans. 1

Sol.



7. Calculate the wavelength (in nanometer) associated with a proton moving at $1.0 \times 10^3 \text{ ms}^{-1}$ (Mass of proton = $1.67 \times 10^{-27} \text{ kg}$ and $h = 6.63 \times 10^{-34} \text{ Js}$) :
 (1) 0.40 nm (2) 2.5 nm (3) 14.0 nm (4) 0.032 nm

Ans. 1

Sol.

As $\lambda = \frac{h}{mv}$

$$\begin{aligned} &= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^3} \\ &= 3.97 \times 10^{-10} \text{ m} \\ &= 0.397 \times 10^{-9} \text{ m} \\ &= \sim 0.40 \text{ nm.} \end{aligned}$$

8. In context with the transition elements, which of the following statements is incorrect ?
- (1) In the highest oxidation states, the transition metal show basic character and form cationic complexes.
 - (2) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4s and 3d electrons are used for bonding.
 - (3) Once the d^5 configuration is exceeded, the tendency to involve all the 3d electrons in bonding decreases.
 - (4) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes.

Ans. 1

Sol.

The basic character of any element changes with the oxidation state, low oxidation states are more basic and high oxidation state are more acidic. For example, MnO and Mn_2O_3 are basic while Mn_2O_7 is acidic in nature.

9. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($h = 6.6 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$, mass of electron, $e_m = 9.1 \times 10^{-31} \text{ kg}$):
- (1) $5.10 \times 10^{-3} \text{ m}$
 - (2) $1.92 \times 10^{-3} \text{ m}$
 - (3) $3.83 \times 10^{-3} \text{ m}$
 - (4) $1.52 \times 10^{-4} \text{ m}$

Ans. 2

Sol.

$$\Delta x \times \Delta P = \frac{h}{4\pi}$$

$$\Delta x \times [m\Delta v] = \frac{h}{4\pi}$$

$$\Delta v = \frac{600 \times 0.005}{100} = 0.03$$

So $\Delta x [9.1 \times 10^{-31} \times 0.03] = \frac{6.6 \times 10^{-34}}{4 \times 3.14}$

$$\Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03} = 1.92 \times 10^{-3} \text{ M.}$$

10. Which of the following pairs represents linkage isomers ?
- (1) $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$
 - (2) $[Co(NH_3)_5NO_3]SO_4$ and $[Co(NH_3)_5(SO_4)]NO_3$
 - (3) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$
 - (4) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$

Ans. 1

Sol.

SCN^- ion can coordinate through the sulphur or nitrogen atom. Such possibilities give rise to linkage isomerism in coordination compounds.

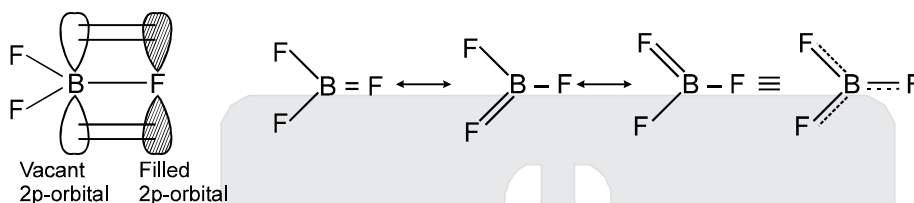
$M \leftarrow SCN$ thiocyanato or thiocyanato-S

$M \leftarrow NCS$ isothiocyanato or thiocyanato-N

11. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1} whereas that of C – F in CF_4 is 515 kJ mol^{-1} . The correct reason for higher B – F bond dissociation energy as compared to that of C – f is :
- (1) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4 .
 - (2) significant $p\pi - p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4 .
 - (3) lower degree of $p\pi - p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4 .
 - (4) smaller size of B - atom as compared to that of C - atom.

Ans. 2

Sol.



Decrease in B – F bond length which results in the higher bond dissociation energy of B – F in BF_3 is due to delocalised $p\pi - p\pi$ bonding between filled p-orbital of F atom and vacant p-orbital of B atom.

Q.12 Using MO theory predict which of the following species has the shortest bond length ?

- (1) O_2^+
- (2) O_2^-
- (3) O_2^{2-}
- (4) O_2^{2+}

Ans. 4

Sol.

Bond Order

- (1) O_2^+ $\frac{10-5}{2} = 2.5$
- (2) O_2^- $\frac{10-7}{2} = 1.5$
- (3) O_2^{2-} $\frac{10-8}{2} = 1$
- (4) O_2^{2+} $\frac{10-4}{2} = 3$

$$\text{Bond order} \propto \frac{1}{\text{bond length}}$$

So, O_2^{2+} has the shortest bond length.

Sol.

In hydrides of 15th group elements the basic strength decreases down the group and the correct order is,
 $\text{SbH}_3 < \text{AsH}_3 < \text{PH}_3 < \text{NH}_3$

18. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statement is correct regarding the behaviour of the solution ?

- (1) The solution is non-ideal, showing +ve deviation from Raoult's Law.
- (2) The solution is non-ideal, showing -ve deviation from Raoult's Law.
- (3) n-heptane shows +ve deviation while ethanol shows -ve deviation from Raoult's Law.
- (4) The solution formed is an ideal solution.

Ans. 1

Sol.

The solution is non-ideal, showing +ve deviation from Raoult's Law.

19. The set representing the correct order of ionic radius is :

- (1) $\text{Na}^+ > \text{Li}^+ > \text{Mg}^{2+} > \text{Be}^{2+}$
- (2) $\text{Li}^+ > \text{Na}^+ > \text{Mg}^{2+} > \text{Be}^{2+}$
- (3) $\text{Mg}^{2+} > \text{Be}^{2+} > \text{Li}^+ > \text{Na}^+$
- (4) $\text{Li}^+ > \text{Be}^{2+} > \text{Na}^+ > \text{Mg}^{2+}$

Ans. 1

Sol.

Down the group ionic radii increases with increasing atomic number because of the increase in the number of atomic shells but across the period the ionic radii decreases due to increase in effective nuclear charge as electrons are added in the same shell. Li^+ and Mg^{2+} are diagonally related, so correct order is $\text{Na}^+ > \text{Li}^+ > \text{Mg}^{2+} > \text{Be}^{2+}$.

$$\text{Be}^{2+} = 0.31 \text{ \AA}$$

$$\text{Mg}^{2+} = 0.72 \text{ \AA}$$

$$\text{Li}^+ = 0.76 \text{ \AA}$$

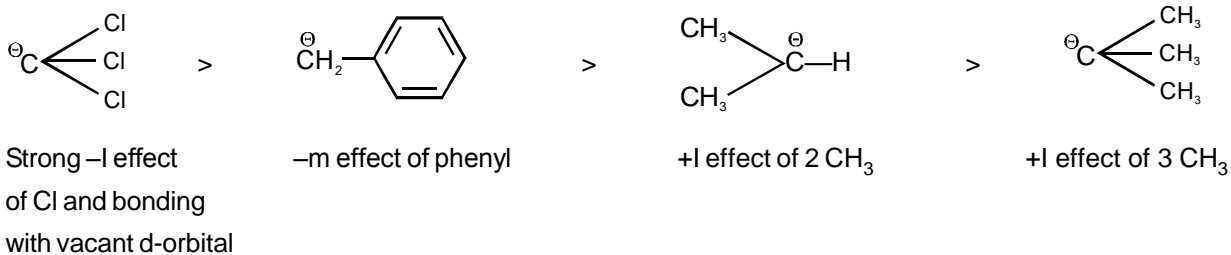
$$\text{Na}^+ = 1.02 \text{ \AA}$$

20. Arrange the carbanions, $(\text{CH}_3)_3\bar{\text{C}}$, $\bar{\text{C}}\text{Cl}_3$, $(\text{CH}_3)_2\bar{\text{C}}\text{H}$, $\text{C}_6\text{H}_5\bar{\text{C}}\text{H}_2$ in order of their decreasing stability :

- (1) $(\text{CH}_3)_2\bar{\text{C}}\text{H} > \bar{\text{C}}\text{Cl}_3 > \text{C}_6\text{H}_5\bar{\text{C}}\text{H}_2 > (\text{CH}_3)_3\bar{\text{C}}$
- (2) $\bar{\text{C}}\text{Cl}_3 > \text{C}_6\text{H}_5\bar{\text{C}}\text{H}_2 > (\text{CH}_3)_2\bar{\text{C}}\text{H} > (\text{CH}_3)_3\bar{\text{C}}$
- (3) $(\text{CH}_3)_3\bar{\text{C}} > (\text{CH}_3)_2\bar{\text{C}}\text{H} > \text{C}_6\text{H}_5\bar{\text{C}}\text{H}_2 > \bar{\text{C}}\text{Cl}_3$
- (4) $\text{C}_6\text{H}_5\bar{\text{C}}\text{H}_2 > \bar{\text{C}}\text{Cl}_3 > (\text{CH}_3)_3\bar{\text{C}} > (\text{CH}_3)_2\bar{\text{C}}\text{H}$

Ans. 2

Sol.



21. Knowing that the Chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statement is incorrect ?

- (1) The ionic sizes of Ln (III) decrease in general with increasing atomic number.
- (2) Ln (III) compounds are generally colourless.
- (3) Ln (III) hydroxides are mainly basic in character
- (4) Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.

Ans. 2

Sol.

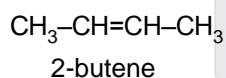
Most of the trivalent lanthanoid compounds except that of La³⁺ and Lu³⁺ are coloured both in the solid state and in the aqueous solution. The colour of these ions can be attributed due to the presence of f-electrons.

22. The alkene that exhibits geometrical isomerism is :

- (1) 2-methyl propene (2) 2-butene (3) 2-methyl-2-butene (4) propene

Ans. 2

Sol.

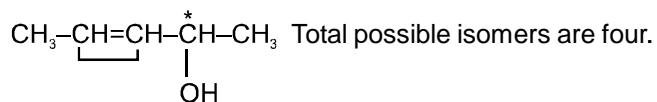


23. The number of stereoisomers possible for a compound of the molecular formula CH₃-CH=CH-CH(OH)-Me is:

- (1) 2 (2) 4 (3) 6 (4) 3

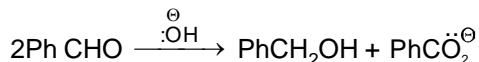
Ans. 2 Where you will be in resonance with IIT-JEE

Sol.



- cis - R
trans - R
cis - S
trans - S

24. In Cannizzaro reaction given below

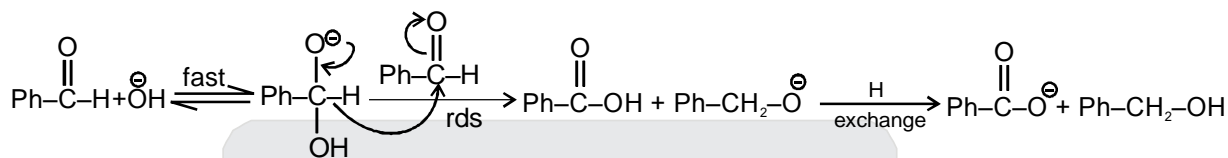


the slowest step is :

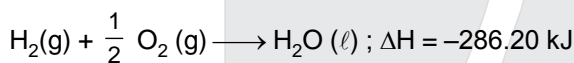
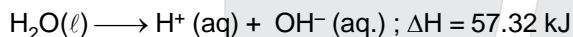
- (1) the transfer of hydride to the carbonyl group (2) the abstraction of proton from the carboxylic group
 (3) the deprotonation of PhCH₂OH (4) the attack of :OH[⊖] at the carboxyl group

Ans. 1

Sol.



25. On the basis of the following thermochemical data : ($\Delta_f G^\circ \text{H}^+_{(\text{aq})} = 0$)

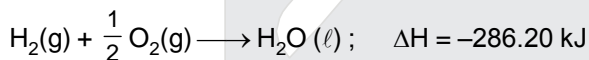


The value of enthalpy of formation of OH⁻ ion at 25°C is :

- (1) -228.88 kJ (2) +228.88 kJ (3) -343.52 kJ (4) -22.88 kJ

Ans. 1

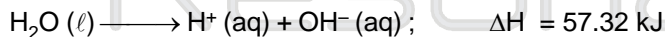
Sol.



$$\Delta H_r = \Delta H_f(\text{H}_2\text{O}, \ell) - \Delta H_f(\text{H}_2, \text{g}) - \frac{1}{2} \Delta H_f(\text{O}_2, \text{g})$$

$$-286.20 = \Delta H_f(\text{H}_2\text{O}(\ell))$$

$$\text{So } \Delta H_f(\text{H}_2\text{O}, \ell) = -286.20$$



$$\Delta H_r = \Delta H_f^\circ(\text{H}^+, \text{aq}) + \Delta H_f^\circ(\text{OH}^-, \text{aq}) - \Delta H_f^\circ(\text{H}_2\text{O}, \ell)$$

$$57.32 = 0 + \Delta H_f^\circ(\text{OH}^-, \text{aq}) - (-286.20)$$

$$\Delta H_f^\circ(\text{OH}^-, \text{aq}) = 57.32 - 286.20 = -228.88 \text{ kJ.}$$

26. Copper crystallises in fcc with a unit cell length of 361 pm. What is the radius of copper atom ?

- (1) 127 pm (2) 157 pm (3) 181 pm (4) 108 pm

Ans. 1

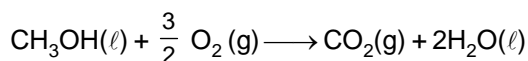
Sol.

In fcc unit cell $4r = \sqrt{2}a$ [r = radius of Cu atom, a = edge length]

$$\text{So } r = \frac{\sqrt{2}a}{4}$$

$$r = \frac{\sqrt{2} \times 361}{4} = 127 \text{ pm.}$$

27. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is

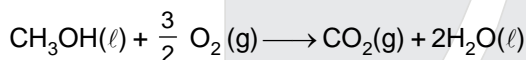


At 298 K standard Gibb's energies of formation for $\text{CH}_3\text{OH}(\ell)$, $\text{H}_2\text{O}(\ell)$ and $\text{CO}_2(\text{g})$ are -166.2 , -237.2 and $-394.4 \text{ kJ mol}^{-1}$ respectively. If standard enthalpy of combustion of methanol is -726 kJ mol^{-1} , efficiency of the fuel cell will be :

- (1) 87% (2) 90% (3) 97% (4) 80%

Ans. 3

Sol.



$$\Delta G_r = \Delta G_f(\text{CO}_2, \text{g}) + 2\Delta G_f(\text{H}_2\text{O}, (\ell)) - \Delta G_f(\text{CH}_3\text{OH}, (\ell)) - \frac{3}{2} \Delta G_f(\text{O}_2, \text{g})$$

$$= -394.4 + 2(-237.2) - (-166.2) - 0$$

$$= -394.4 - 474.4 + 166.2$$

$$= -868.8 + 166.2$$

$$\Delta G_r = -702.6 \text{ kJ}$$

$$\% \text{ efficiency} = \frac{702.6}{726} \times 100 = 97\%.$$

28. Two liquids X and Y form an ideal solution. At 300K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mmHg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mmHg. Vapour pressure (in mmHg) of X and Y in their pure states will be, respectively :

- (1) 300 and 400 (2) 400 and 600 (3) 500 and 600 (4) 200 and 300

Ans. 2

Sol.

$$P_A^{\circ} X_A + P_B^{\circ} X_B = P_{\text{total}}$$

$$= P_A^{\circ} \times \frac{1}{4} + P_B^{\circ} \times \frac{3}{4} = 550$$

$$= P_A^{\circ} + 3P_B^{\circ} = 550 \times 4 \quad \dots(i)$$

similarly

$$= P_A^{\circ} \times \frac{1}{5} + P_B^{\circ} \times \frac{4}{5} = 560$$

$$P_A^{\circ} + 4P_B^{\circ} = 560 \times 5 \quad \dots(ii)$$

eq. (ii) – eq.(i)

$$P_B^{\circ} = 560 \times 5 - 550 \times 4 = 600$$

so $P_A^{\circ} = 400.$

29. Given :

$$E_{\text{Fe}^{3+}/\text{Fe}}^{\circ} = -0.036\text{V}, E_{\text{Fe}^{2+}/\text{Fe}}^{\circ} = -0.439\text{V}$$

The value of standard electrode potential for the change, $\text{Fe}_{(\text{aq})}^{3+} + e^{-} \longrightarrow \text{Fe}^{2+}(\text{aq})$ will be :

(1) 0.385V

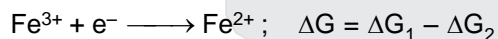
(2) 0.770V

(3) -0.270V

(4) -0.072V

Ans. 2

Sol.



$$\Delta G = 3 \times 0.036F - 2 \times 0.439 \times F = -1 \times E^{\circ}(\text{Fe}^{3+}/\text{Fe}^{2+}) \times F$$

$$E^{\circ}(\text{Fe}^{3+}/\text{Fe}^{2+}) = 2 \times 0.439 - 3 \times 0.036$$

$$= 0.878 - 0.108$$

$$= 0.770\text{V}$$

30. The half life period of a first order chemical reaction is 6.93 minutes. Time required for the completion of 99% of the chemical reaction will be ($\log 2 = 0.301$) :

(1) 23.03 minutes

(2) 46.06 minutes

(3) 460.6 minutes

(4) 230.3 minutes

Ans. 2

Sol.

In first order reaction for X% completion

$$k = \frac{2.303}{t} \log \left(\frac{100}{100 - x\%} \right)$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log \left(\frac{100}{100 - 99} \right)$$

$$= \frac{0.693}{6.93} = \frac{2.303 \times 2}{t}$$

So, $t = 46.06$ min.

PART-B (MATHEMATICS)

Direction : Question number 31 to 35 are Assertion-Reason type questions. Each of these questions contains two statements :

Statement - 1 (Assertion) and

Statement - 2 (Reason)

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

31. **Statement - 1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement - 2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. 2

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Sol.

Statement-1 is true

Statement-2 is false

32. Let A be a 2×2 matrix

Statement-1 : $\text{adj}(\text{adj} A) = A$

Statement-2 : $|\text{adj} A| = |A|$

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. 1

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\text{Adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\text{adj}(\text{adj} A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$$

Statement-1 is true

$$|A| = ad - bc \quad \text{and} \quad |\text{adj} A| = ad - bc$$

\therefore Statement-1 is true,

statement-2 is also correct but does not explain statement-1

33. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$.

Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement-2 : f is a bijection.

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1

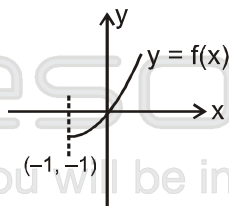
(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. 2

Sol. f is one-one



it is clear that $f(x) = f^{-1}(x)$ has some solution as $f(x) = x \Rightarrow (x + 1)^2 - 1 = x$

$$\Rightarrow x = -1, 0$$

Now co-domain of f is not given so we consider co-domain of f as \mathbb{R} so f is not onto

34. **Statement-1** : The variance of first n even natural numbers is $\frac{n^2 - 1}{4}$

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural

numbers is $\frac{n(n+1)(2n+1)}{6}$

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. 3

Sol. $2^2 + 4^2 + 6^2 + \dots + (2n)^2 = 4(1^2 + 2^2 + \dots + n^2) = 4 \cdot \frac{n(n+1)(2n+1)}{6}$

$2 + 4 + 6 + \dots + 2n$ or $2(1 + 2 + \dots + n) = n(n+1)$

\therefore variance = $\frac{2}{3}(n+1)(2n+1) - (n+1)^2 = \frac{(n+1)(4n+2-3n-3)}{3} = \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

\therefore Statement-1 is false

Statement-2 is true

35. Let $f(x) = x|x|$ and $g(x) = \sin x$

Statement-1 : $g \circ f$ is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : $g \circ f$ is twice differentiable at $x = 0$

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Ans. 2

Sol. $g \circ f(x) = \sin(x|x|)$

$$g \circ f(x) = \begin{cases} \cos(x|x|)(|x| + x \cdot \frac{x}{|x|}) & , x \neq 0 \\ |x| \cos(x|x|) & , x = 0 \end{cases}$$

\therefore $g \circ f$ is differentiable at $x = 0$ and the derivatives is continuous

\therefore statement-1 is true

38. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(1) $\frac{2\sqrt{3}}{8}$

(2) $\frac{3\sqrt{2}}{5}$

(3) $\frac{\sqrt{3}}{4}$

(4) $\frac{3\sqrt{2}}{8}$

Ans. 4

Sol. as shortest distance is along the common normal so $\frac{dy}{dx}$ of both the curves must be equal

$$y = x + 1 \quad \Rightarrow \quad y' = 1$$

$$\text{and } x = y^2 \quad \Rightarrow \quad 1 = 2yy' \quad \Rightarrow \quad y = \frac{1}{2}$$

$$P \left(\frac{1}{4}, \frac{1}{2} \right)$$

distance from P to the line $y = x + 1$ is $\frac{3\sqrt{2}}{8}$

39. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

(1) $(-6, 7)$

(2) $(5, -15)$

(3) $(-5, 5)$

(4) $(6, -17)$

Ans. 1

Sol. Parallel vector of line and normal vector of plane are \perp

$$3 + (-15) - 2\alpha = 0$$

$$\alpha = -6$$

$$\text{Now } 2 + 3 - \alpha(-2) + \beta = 0 \quad [(2, 1, -2) \text{ lies on the plane}]$$

$$\Rightarrow \beta = -5 + 12 = 7$$

40. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is:

(1) at least 500 but less than 750

(2) at least 750 but less than 1000

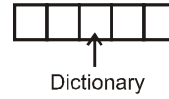
(3) at least 1000

(4) less than 500

Ans. 3

Sol. 6 different novels and 3 different dictionaries

\therefore number of ways = ${}^3C_1 \times {}^6C_4 \times 4! = 1080$



41. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to

$\frac{9}{10}$, then n is greater than :

- (1) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (2) $\frac{9}{\log_{10} 4 - \log_{10} 3}$ (3) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (4) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

Ans. 4

Sol. By binomial distribution, $\left(\frac{1}{4} + \frac{3}{4}\right)^n$

\therefore probability of at least one success = $1 - \text{no. of success} = 1 - {}^nC_n \left(\frac{3}{4}\right)^n = 1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{10}$

Taking \log_{10} on both sides

$n(\log_{10} 3 - \log_{10} 4) \leq -1$

or $n \geq \frac{-1}{\log_{10} 3 - \log_{10} 4} \Rightarrow n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

42. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for:

- (1) exactly one value of p (2) exactly two values of p
 (3) more than two values of p (4) no value of p

Ans. 1

Sol. $p(p^2 + 1)x - y + q = 0$

$(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular

for a common line

\Rightarrow lines are parallel

\Rightarrow slopes are equal

$\therefore \frac{p(p^2 + 1)}{1} = -\frac{(p^2 + 1)^2}{(p^2 + 1)} \Rightarrow p = -1$

43. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
 (1) $A = C$ (2) $B = C$ (3) $A \cap B = \phi$ (4) $A = B$

Ans. 2

Sol. Let $x \in B$

case-1 $x \in A \Rightarrow x \in A \cap B \Rightarrow x \in A \cap C \quad (\because A \cap B = A \cap C)$
 $\Rightarrow x \in C$

Case-2 $x \notin A \Rightarrow x \in A \cup B \Rightarrow x \in A \cup C \quad (\because A \cup B = A \cup C)$
 $\Rightarrow x \in C$

Hence $x \in B \Rightarrow x \in C \Rightarrow B \subseteq C \rightarrow (i)$

Similarly we can prove $C \subseteq B \rightarrow (ii)$

By (i), (ii), $B = C$

44. For real x, let $f(x) = x^3 + 5x + 1$, then
 (1) f is onto R but not one-one (2) f is one-one and onto R
 (3) f is neither one-one nor onto R (4) f is one-one but not onto R

Ans. 2

Sol. $f(x) = x^3 + 5x + 1$
 $f'(x) = 3x^2 + 5$
 Hence function is SI function
 hence one-one and onto

45. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constant, is
 (1) $y'' = y'y$ (2) $yy'' = y'$ (3) $yy'' = (y')^2$ (4) $y' = y^2$

Ans. 3

Sol. $y = c_1 e^{c_2 x}$
 $y' = c_1 c_2 e^{c_2 x} \Rightarrow c_2 = \frac{y'}{y}$

$y'' = c_1 c_2^2 e^{c_2 x} \Rightarrow y'' = y \cdot \left(\frac{y'}{y}\right)^2$
 $\Rightarrow yy'' = (y')^2$

46. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

then the value of n is :

- (1) any even integer (2) any odd integer (3) any integer (4) Zero

Ans. 2

Sol. As $ab + bc \neq 0$

$$\text{Now } \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ -2 & -2 & 2 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & 1 & 0 \\ -b & 2b+1 & 4b \\ c & -1 & 0 \end{vmatrix} + \begin{vmatrix} a+c & b+c & c-1 \\ 0 & 0 & 2 \\ (-1)^n(a+c) & (-1)^n(c-b) & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow -4b(-a-c) - 2[(-1)^n(c-b)(a+c) - (-1)^n(b+c)(a+c)] = 0$$

$$\Rightarrow 4b(a+c) - 2(-1)^n[(c-b)(a+c) - (b+c)(a+c)] = 0$$

$$\Rightarrow 4b(a+c) - 2(-1)^n(a+c)[c-b-b-c] = 0$$

$$\Rightarrow 4b(a+c) + 4b(-1)^n(a+c) = 0$$

$$\Rightarrow n \text{ is odd integer}$$

47. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is :

- (1) 2 (2) 7 (3) 8 (4) 0

Ans. 1

$$\text{Sol. } (9 - 1)^{2n} - (63 - 1)^{2n+1} = [{}^{2n}C_0 9^{2n} + {}^{2n}C_1 \cdot 9^{2n-1}(-1) + \dots + {}^{2n}C_{2n-1} 9(-1)^{2n-1} + 1] - [{}^{2n+1}C_0 (63)^{2n+1} - {}^{2n+1}C_1 (63)^{2n} + \dots - 1]$$

$$\therefore \text{Remainder} = 2$$

48. Let y be an implicit function of x-defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y' (1) equals :

- (1) 1 (2) $\log 2$ (3) $-\log 2$ (4) -1

Ans. 4

Sol. $x^{2x-1} \cdot 2x + 2x^{2x} \ln x - 2(x^x \ln x + x \cdot x^{x-1}) \cot y + 2x^x \operatorname{cosec}^2 y \cdot y' = 0$

$y = \frac{\pi}{2}$ and $x = 1$

$\Rightarrow 2 + 0 - 0 + 2y' = 0$
 $y' = -1$

49. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x , the expression $3b^2 x^2 + 6bcx + 2c^2$ is :

- (1) less than $4ab$ (2) greater than $-4ab$ (3) less than $-4ab$ (4) greater than $4ab$

Ans. 2

Sol. $bx^2 + cx + a = 0$
 Roots are imaginary
 $\therefore c^2 - 4ab < 0$

Now minimum value of $3b^2x^2 + 6bcx + 2c^2$ is equal to $-\frac{(36b^2c^2 - 4 \cdot 3b^2 \cdot 2c^2)}{4 \cdot (3b^2)} = \frac{-12b^2c^2}{12b^2} = -c^2$

which is greater than $-4ab$

50. The sum to infinity of the series

$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$ is :

- (1) 3 (2) 4 (3) 6 (4) 2

Ans. 1

Sol. $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \infty$

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots \infty$

$\frac{2}{3}S = \left(1 + \frac{1}{3}\right) + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots \infty$

$\frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} \Rightarrow \frac{2}{3}S = 2 \Rightarrow S = 3$

51. The projections of a vector on the three coordinate axis are 6, -3, 2 respectively. The direction cosines of the vector are :

- (1) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (2) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (3) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (4) 6, -3, 2

Ans. 2

Sol. $lr = 6, mr = -3, nr = 2$
 $\therefore r^2 (\ell^2 + m^2 + n^2) = 36 + 9 + 4 = 49$
 $\Rightarrow r = 7$
 $\langle \ell, m, n \rangle \equiv \langle \frac{6}{7}, \frac{-3}{7}, \frac{2}{7} \rangle$

52. Let A and B denote the statements

$$A : \cos \alpha + \cos \beta + \cos \gamma = 0$$

$$B : \sin \alpha + \sin \beta + \sin \gamma = 0$$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

- (1) A is false and B is true (2) both A and B are true
 (3) both A and B are false (4) A is true and B is false

Ans. 2

Sol. $2\{\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)\} + 3 = 0$
 $(\cos \alpha + \cos \beta + \cos \gamma)^2 + (\sin \alpha + \sin \beta + \sin \gamma)^2 = 0$
 $\sum \cos \alpha = 0 = \sum \sin \alpha$

53. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equal :

- (1) $\frac{1}{7}$ (2) $\frac{5}{14}$ (3) $\frac{1}{50}$ (4) $\frac{1}{14}$

Ans. 4

Sol. Favourable case ; 08
 Total cases : 00, 01, 02, ..., 09, 10, 20, 30, 40
 Required probability = $\frac{1}{14}$

54. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (-1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point :

- (1) $\left(\frac{5}{4}, 0\right)$ (2) $\left(\frac{5}{2}, 0\right)$ (3) $\left(\frac{5}{3}, 0\right)$ (4) 0, 0

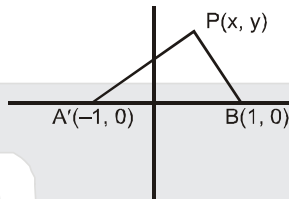
Ans. 1

Sol. $\therefore \frac{PA'}{PB'} = \frac{3}{1}$

$$\begin{aligned} \therefore (x+1)^2 + y^2 &= 9((x-1)^2 + y^2) \\ x^2 + 2x + 1 + y^2 &= 9x^2 + 9y^2 - 18x + 9 \\ 8x^2 + 8y^2 - 20x + 8 &= 0 \end{aligned}$$

$$x^2 + y^2 - \frac{10}{4}x + 1 = 0$$

\therefore circumcentre $\left(\frac{5}{4}, 0\right)$



55. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then the d is equal to :
- (1) 20.0 (2) 10.1 (3) 20.2 (4) 10.0

Ans. 2

Sol. 1, 1 + d, 1 + 2d 1 + 100d

$$\text{mean} = \frac{101+1}{2} = 51\text{st term} \Rightarrow 1 + 50d$$

$$\text{mean deviation} = \frac{\sum |x_i - \mu|}{n}$$

$$225 = \frac{2 \times \frac{50 \times 51}{2} d}{101}$$

$$d = 10.1$$

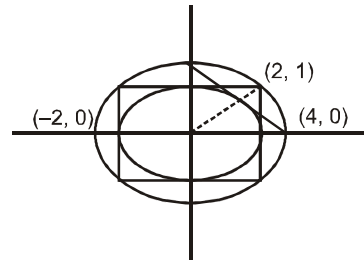
56. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle alingent with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is :
- (1) $x^2 + 12y^2 = 16$ (2) $4x^2 + 48y^2 = 48$ (3) $4x^2 + 64y^2 = 48$ (4) $x^2 + 16y^2 = 16$

Ans. 1

Sol. Equation of ellipse $\frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$ is passes through (2, 1)

$$\therefore \frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow \frac{1}{b^2} = \frac{12}{16}$$

equation of required ellipse $\frac{x^2}{16} + \frac{12y^2}{16} = 1$



57. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to :

(1) $\sqrt{5} + 1$

(2) 2

(3) $2 + \sqrt{2}$

(4) $\sqrt{3} + 1$

Ans. 1

Sol. $\left| \left| z - \frac{4}{z} \right| - \frac{4}{|z|} \right| \leq \left| z - \frac{4}{z} + \frac{4}{|z|} \right| \leq \left| z - \frac{4}{z} \right| + \frac{4}{|z|}$

$$|z| \leq 2 + \frac{4}{|z|}$$

$$r \leq 2 + \frac{4}{r}$$

$$r^2 - 2r - 4 \leq 0$$

$$r = 1 \pm \sqrt{5}$$

58. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for :

(1) all except one value of p

(2) all except two values of p

(3) exactly one value of p

(4) all values of p

Ans. 1

Sol. $S_1 + \lambda S_2 = 0$ should satisfy (1, 1)

$$(2 + 3 + 7 + 2p - 5) + \lambda (1 + 1 + 2 + 2 - p^2) = 0$$

$$\lambda = -\frac{7 + 2p}{6 - p^2}$$

$$p^2 \neq 6 \Rightarrow p \neq \pm \sqrt{6}$$

but at $p = \pm \sqrt{6}$ the 2nd circle is

$$x^2 + y^2 + 2x + 2y - 6 = 0$$

satisfies (1, 1) and obviously P and Q

so $p = \pm \sqrt{6}$ is also acceptable

$$\lambda \neq -1 \Rightarrow \frac{7+2p}{6-p^2} \neq 1 \Rightarrow 7+2p \neq 6-p^2$$

$$p^2 + 2p + 1 \neq 0$$

$$p \neq -1$$

59. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality

$$[3\vec{u} \ p\vec{v} \ p\vec{w}] - [p\vec{v} \ \vec{w} \ q\vec{u}] - [2\vec{w} \ q\vec{v} \ q\vec{u}] = 0$$

holds for :

(1) exactly two values of (p, q)

(2) more than two but not all values of (p, q)

(3) all values of (p, q)

(4) exactly one values of (p, q)

Ans. 4

Sol. $3p^2 - pq + 2q^2 = 0$
 $\Rightarrow p = 0, q = 0$

60. $\int_0^{\pi} [\cot x] dx$, where $[\cdot]$ denotes the greatest integer function, equal to :

(1) 1

(2) -1

(3) $-\frac{\pi}{2}$

(4) $\frac{\pi}{2}$

Ans. 3

Sol. $I = \int_0^{\pi} [\cot x] dx$

$$\Rightarrow I = \int_0^{\pi} [\cot(\pi - x)] dx \Rightarrow I = \int_0^{\pi} [-\cot x] dx$$

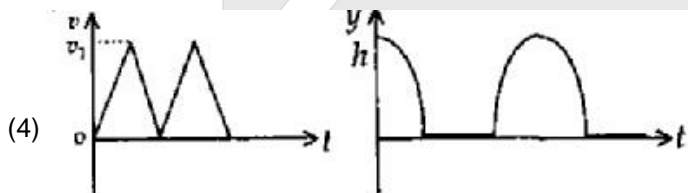
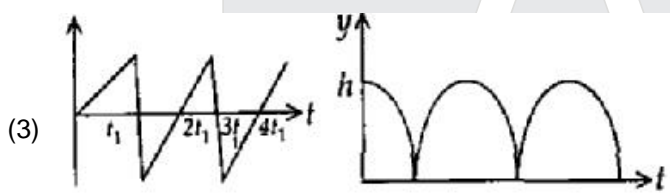
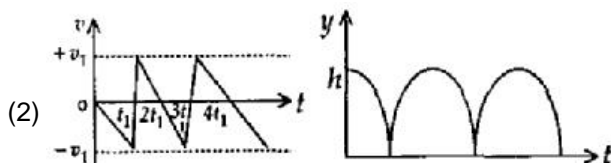
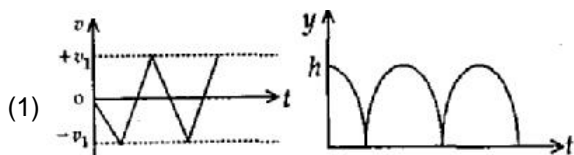
$$\Rightarrow 2I = \int_0^{\pi} ([\cot x] + [-\cot x]) dx = \int_0^{\pi} -1 dx$$

$$\Rightarrow 2I = -\pi$$

$$\Rightarrow I = -\frac{\pi}{2}$$

PART - C (PHYSICS)

61. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be:



Ans. 2

Sol.

Height is taken as positive in upward direction. Initial velocity of ball is in downward direction i.e., initial velocity is negative.

62. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R , the radius of the earth, is

(1) $\frac{R}{\sqrt{2}}$

(2) $\frac{R}{2}$

(3) $\sqrt{2} R$

(4) $2R$

Ans. 4

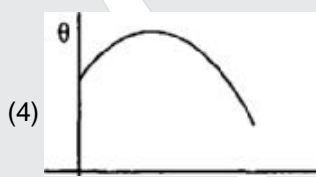
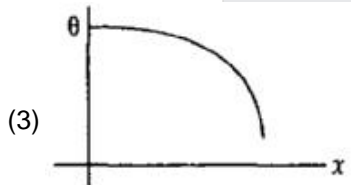
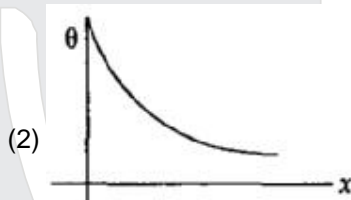
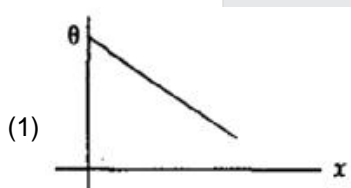
Sol.

Acceleration due to gravity at height h from earth surface.

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$\frac{g}{9} = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \quad \Rightarrow \quad h = 2R$$

63. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures



Ans. 1

Sol.

$$\frac{dQ}{dt} = -kA \frac{d\theta}{dx}$$

at steady state $\frac{dQ}{dt} = \text{constant}$.

$$d\theta \propto -dx$$

$$\int_{\theta_0}^{\theta} d\theta = -k \int_0^x dx$$

$$\theta = \theta_0 - kx$$



64. Two points P and Q are maintained at the potentials of 10 V and -4 V respectively. The work done in moving 100 electrons from P to Q is :

- (1) 9.60×10^{-17} J (2) -2.24×10^{-16} J (3) 2.24×10^{-16} J (4) -9.60×10^{-17} J

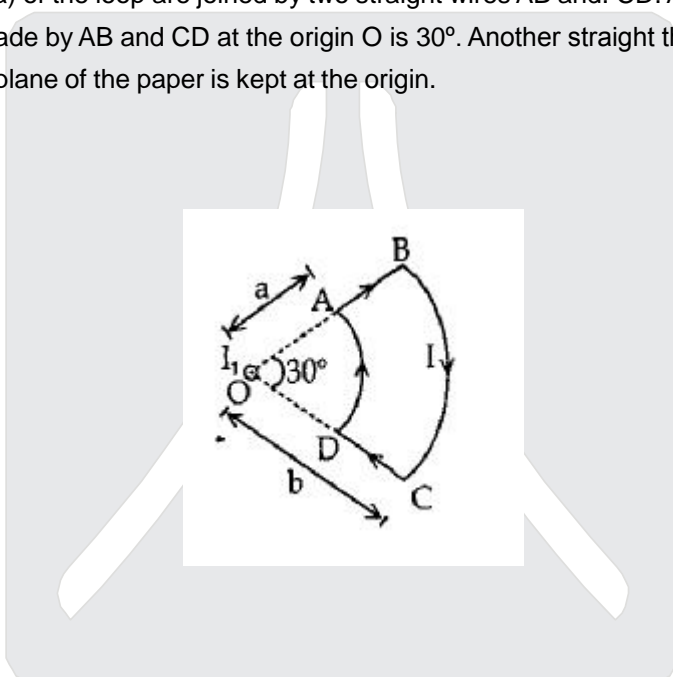
Ans. 3

Sol.

$$\begin{aligned} W_{P \rightarrow Q} |_{\text{ext}} &= q (V_Q - V_P) \\ &= -1.6 \times 10^{-19} \times 100 (-4 - 10) \\ &= 2.24 \times 10^{-16} \text{ J} \end{aligned}$$

Direction : Question numbers 65 and 66 are based on the following paragraph :

A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30° . Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



65. The magnitude of the magnetic field (B) due to the loop ABCD at the origin (O) is:

- (1) $\frac{\mu_0 I (b-a)}{24ab}$ (2) $\frac{\mu_0 I}{4\pi} \left[\frac{b-a}{ab} \right]$
 (3) $\frac{\mu_0 I}{4\pi} \left[2(b-a) + \frac{\pi}{3}(a+b) \right]$ (4) zero

Ans. 1

Sol.

Magnetic field (B) due to loop ABCD

$$\begin{aligned} &= \frac{\mu_0 I}{4\pi} \left(\frac{\pi}{6} \right) \times \left[\frac{1}{a} - \frac{1}{b} \right] \\ &= \frac{\mu_0 I}{24} \left[\frac{b-a}{ab} \right] \end{aligned}$$

66. Due to the presence of the current I_1 at the origin :

(1) The forces on AD and BC are zero.

(2) The magnitude of the net force on the loop is given by $\frac{\mu_0 I_1 I}{4\pi} \left[2(b-a) + \frac{\pi}{3}(a+b) \right]$

(3) the magnitude of the net force on the loop is given by $\frac{\mu_0 I_1 I}{24ab} (b-a)$.

(4) the forces on AB and DC are zero.

Ans. 1

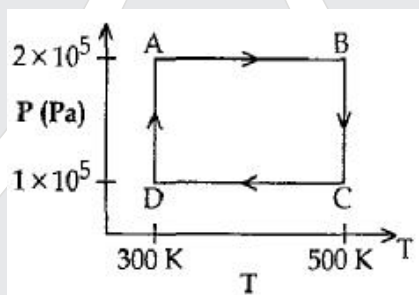
Sol. $\vec{F} = i (\vec{\ell} \times \vec{B})$

Magnetic field due to I_1 is parallel to AD and BC. So that force On AD and BC is zero.

Directions : Question number 67, 68 and 69 are based on the following paragraph.

Paragraph :

Two moles of helium gas are taken over the cycle ABCDA, as shown in the P-T diagram.



67. Assuming the gas to be ideal the work done on the gas in taking it from A to B is :

- (1) 300 R (2) 400 R (3) 500 R (4) 200 R

Ans. 2

Sol.

n (moles) = 2

A to B is isobaric process

$$W_{AB} = P\Delta V = nR\Delta T$$

$$= (2) (R) (200)$$

$$= 400 R$$

$$W_{AB} = 400 R$$

68. The work done on the gas in taking it from D to A is

- (1) +414 R (2) - 690 R (3) + 690 R (4) - 414 R

Ans. 1

Sol.

D to A is isothermal process

Work done by the gas in D to A is

$$\begin{aligned}W_{DA} &= nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2} \\&= (2) (R) (300) \ln \frac{10^5}{2 \times 10^5} \\&= (600 R) [-\ln 2] \\&= -(600 R) (0.693) \\&= -414 R\end{aligned}$$

$W_{DA} = -414 R$, it is work done by the gas

So work done on the gas is + 414 R

69. The net work done on the gas in the cycle ABCDA is:

(1) 276 R

(2) 1076 R

(3) 1904 R

(4) Zero

Ans. 1

Sol.

$$\begin{aligned}W_{ABCD A} &= W_{AB} + W_{BC} + W_{CD} + W_{DA} \\&= nR (\Delta T)_{AB} + nR (T_B) \ln \frac{P_B}{P_C} \\&\quad + nR (\Delta T)_{CD} + nR (T_D) \ln \frac{P_D}{P_A} \\&= nR (200) + 500 nR \ln 2 + nR (-200) \\&\quad + 300 R \ln \frac{1}{2} \\&= 2 \ln 2 [500 R - 300 R] \\&= (400 R) (\ln 2) = (400 R) (0.693) = 276 R \\W_{ABCD A} &= 276 R\end{aligned}$$

70. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (0.5°), then the least count of the instrument is:

(1) half minute

(2) one degree

(3) half degree

(4) one minute

Ans. 4

Sol.

29 division of main scale coincides with 30 divisions of vernier scale Hence one division of vernier scale

$$\begin{aligned}&= \frac{30 - 29}{30} \text{ of main scale} = \frac{1}{30} \times 0.5^\circ \\&= \frac{1}{30} \times 0.5 \times 60 \text{ min} = 1 \text{ min.}\end{aligned}$$

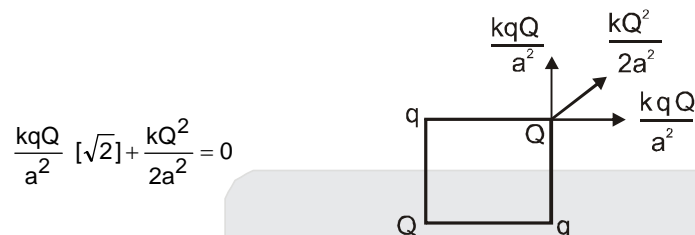
71. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals

- (1) -1 (2) 1 (3) $-\frac{1}{\sqrt{2}}$ (4) $-2\sqrt{2}$

Ans. 4

Sol.

Since F_{net} is zero



$$\frac{kqQ}{a^2} [\sqrt{2}] + \frac{kQ^2}{2a^2} = 0$$

$$\frac{Q}{q} = -2\sqrt{2}$$

72. One kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ N/m}^2$. The density of the gas is 4 kg/m^3 . What is the energy of the gas due to its thermal motion?

- (1) $5 \times 10^4 \text{ J}$ (2) $6 \times 10^4 \text{ J}$ (3) $7 \times 10^4 \text{ J}$ (4) $3 \times 10^4 \text{ J}$

Ans. 1

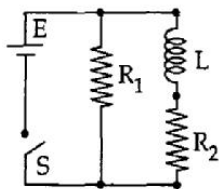
Sol.

$$\frac{m}{d} = \frac{1}{4} = m^3$$

kE = for diatomic

$$kE = \frac{5}{2} PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

73.

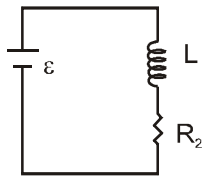


An inductor of inductance $L = 400 \text{ mH}$ and resistors of resistances $R_1 = 2 \Omega$ and $R_2 = 2 \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is :

- (1) $\frac{12}{t} e^{-3t} \text{ V}$ (2) $6(1 - e^{-t/0.2}) \text{ V}$ (3) $12 e^{-5t} \text{ V}$ (4) $6 e^{-5t} \text{ V}$

Ans. 3

Sol.



$$V_L = \varepsilon e^{-\frac{R_2 t}{L}} = 12. e^{-\frac{2t}{400 \times 10^{-3}}} = 12 e^{-5t}$$

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

74. **Statement 1** : The temperature dependence of resistance is usually given as $R = R_0 (1 + \alpha \Delta t)$. The resistance of a wire changes from 100Ω to 150Ω when its temperature is increased from 27°C to 227°C . This implies that $\alpha = 2.5 \times 10^{-3}/^\circ\text{C}$.
- Statement 2** : $R = R_0 (1 + \alpha \Delta t)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) \ll R_0$.
- (1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

Ans. 3

Sol.

$$\alpha = \frac{R - R_0}{\Delta t} = \frac{150 - 100}{200} = 0.25/^\circ\text{C}$$

Statement 1 is wrong Statement 2 is correct

75. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from:
- (1) $3 \rightarrow 2$ (2) $4 \rightarrow 2$ (3) $5 \rightarrow 4$ (4) $2 \rightarrow 1$

Ans. 3

Sol.

$5 \rightarrow 4$ Transition energy from 5 to 4 will be less than from $4 \rightarrow 3$. All other transition energy are higher than that for $4 \rightarrow 3$.

76. A mixture of light consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is:
- (1) 885.0 nm (2) 442.5 nm (3) 776.8 nm (4) 393.4 nm

Ans. 2

Sol.

$$3 \frac{D\lambda k}{d} = 4 \frac{D\lambda u}{d}$$

$$\Rightarrow \lambda_u = \frac{3\lambda k}{4} = \frac{3}{4} \times 590 \text{ nm}$$

$$= 442.5 \text{ nm}$$

77. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is :

(1) $7\sqrt{2}$ units

(2) 7 units

(3) 8.5 units

(4) 10 units

Ans. 1

Sol.

$$\vec{v} = \vec{u} + \vec{a} t$$

$$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

$$= 7\hat{i} + 7\hat{j}$$

$$|\vec{v}| = 7\sqrt{2}$$

78. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is : ($hc = 1240 \text{ eV}\cdot\text{nm}$)

(1) 1.41 eV

(2) 1.51 eV

(3) 1.68 eV

(4) 3.09 eV

Ans. 1

Sol.

$$E_\lambda = \frac{1240}{400} \text{ eV} = 3.1 \text{ eV}$$

$$E_\lambda - k = (3.10 - 1.68) \text{ eV}$$

$$= 1.42 \text{ eV}$$

$$Q < 1.42 \text{ eV}$$

79. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν , $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be:

(1) 3

(2) 2

(3) 1

(4) 4

Ans. 3

Sol.

$$\text{Number of beats produced} = 1$$

80. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2 . At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1})

(1) 98 m

(2) 147 m

(3) 196 m

(4) 49 m

Ans. 1

Sol.

Motor cycle has travelled a distance s . Its velocity at that point

$$v = \sqrt{2as}$$

The observed frequency

$$f = f \frac{330 - v}{330}$$

$$\Rightarrow 0.94 = \frac{330 - v}{330}$$

$$\Rightarrow v = 0.06 \times 330 \text{ m/s} \\ = 19.8 \text{ m/s}$$

$$s = \frac{v^2}{2a} = \frac{19.8^2}{2 \times 2} = 9.9^2 = 98 \text{ m}$$

81. The above is a plot of binding energy per nucleon E_b , against the nuclear mass M ; A, B, C, D, E, correspond to different nuclei. Consider four reactions :

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$ (iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$,

where ε is the energy released? In which reactions is ε positive?

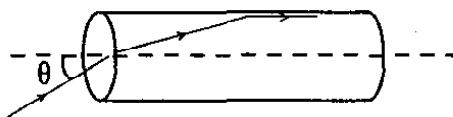
(1) (i) and (iii) (2) (ii) and (iv) (3) (ii) and (iii) (4) (i) and (iv)

Ans. 4

Sol.

If binding energy of product nuclei is greater then energy is released.

82. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

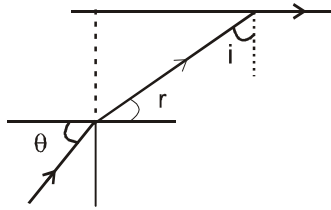


The incident angle (θ) for which the light ray grazes along the wall of the rod is:

(1) $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$ (2) $\sin^{-1} \left(\frac{2}{\sqrt{3}} \right)$ (3) $\sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$ (4) $\sin^{-1} \left(\frac{1}{2} \right)$

Ans. 3

Sol.



$$\sin \theta = \frac{2}{\sqrt{3}} \sin r = \frac{2}{\sqrt{3}} \cos i \quad \dots(i)$$

$$\text{and } \frac{2}{\sqrt{3}} \sin i = \sin 90^\circ \Rightarrow i = 60^\circ \quad \dots(ii)$$

From (i) and (ii)

$$\sin \theta = \frac{1}{\sqrt{3}}$$

83. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F , how much force is needed to stretch wire 2 by the same amount?

- (1) $4F$ (2) $6F$ (3) $9F$ (4) F

Ans. 3

Sol.

$$F = \frac{Y A x}{\ell}$$

$$\text{and } F_2 = \frac{Y(3A)x}{(\ell/3)} = 9F$$

This question contains Statement—1 and Statement—2. Of the four choices given after the statements, choose the one that best describes the two statements.

84. **Statement 1** : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement 2 : The net work done by a conservative force on an object moving along a closed loop is zero.

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

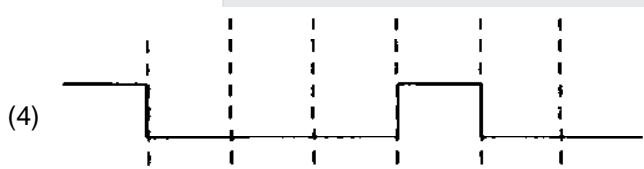
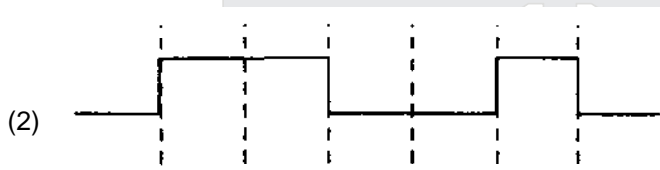
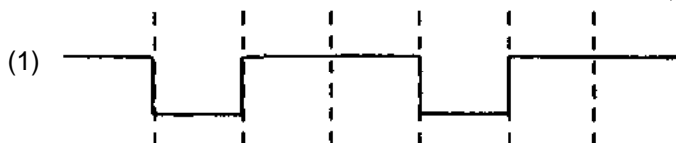
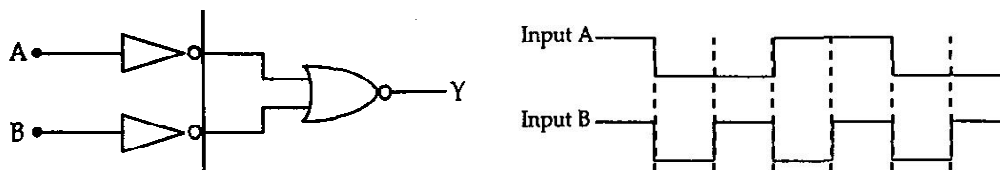
Ans. 1

Sol.

Statement—1 : at correct as the field is conservative

statement —2 : correct Explanation

85. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct out put waveform.



Ans. 4 Where you will be in resonance with IIT-JEE

Sol.

$$Y = \overline{(\overline{A} + \overline{B})} = A.B$$

it is AND gate.

86. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T , then, which of the following does not change with time?

(1) $\frac{aT}{x}$ (2) $aT + 2\pi v$ (3) $\frac{aT}{x}$ (4) $a^2T^2 + 4\pi^2v^2$

Ans. 1 or 4

Sol. $x = A \sin \omega t$
 $v = A \omega \cos \omega t$
 $a = -A\omega^2 \sin \omega t$

(1) $\frac{aT}{x} = \frac{-A\omega^2 \sin \omega t}{A \sin \omega t} = -\omega^2$

(2) $aT + 2\pi v = -A\omega^2 T \sin \omega t + 2\pi A\omega \cos \omega t$

(3) $\frac{aT}{v} = \frac{-A\omega^2 \sin \omega t \times T}{A\omega \cos \omega t}$

(4) $a^2T^2 + 4\pi^2 v^2 = + A^2 \omega^4 T^2 \sin^2 \omega t + 4\pi^2 A^2 \omega^2 \cos^2 \omega t = 4\pi^2 A^2 \omega^2$

87. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of :

(1) $\frac{1}{6} \frac{\ell \omega}{g}$ (2) $\frac{1}{2} \frac{\ell^2 \omega^2}{g}$ (3) $\frac{1}{6} \frac{\ell^2 \omega^2}{g}$ (4) $\frac{1}{3} \frac{\ell^2 \omega^2}{g}$

Ans. 3

Sol. $\frac{1}{2} I\omega^2 = mgh$

$\frac{1}{2} \left(\frac{m\ell^2}{3} \right) \omega^2 = mgh$

$h = \frac{\omega^2 \ell^2}{6g}$

88. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v , from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be:

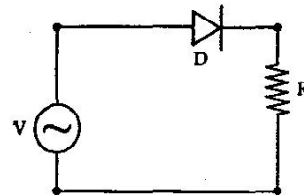
(1) $\left(\frac{f}{2}, \frac{f}{2} \right)$ (2) (f, f) (3) $(4f, 4f)$ (4) $(2f, 2f)$

Ans. 4

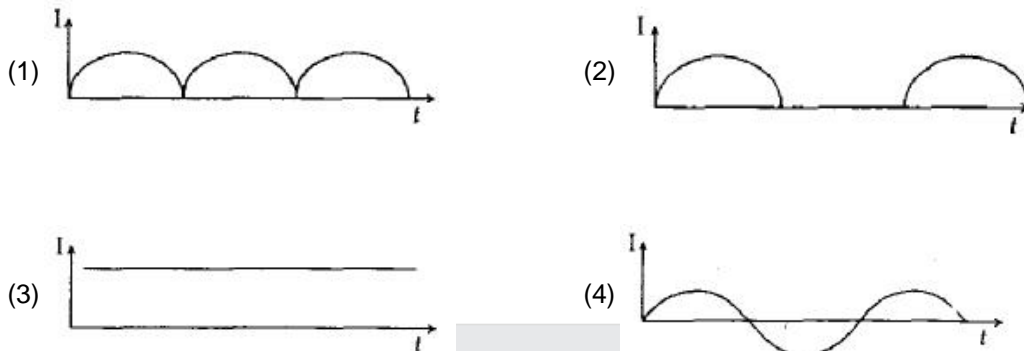
Sol.

$v = u$ and $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $\frac{2}{v} = \frac{1}{f} \Rightarrow v = 2f, u = 2f$

89. A p-n junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit.



The current (I) in the resistor (R) can be shown by:



Ans. 2

Sol.

From Half wave rectifier

90. Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of sphere, the magnitude of electric field is:

- (1) $\frac{Q}{4\pi\epsilon_0 r_1^2}$ (2) $\frac{Qr_1^2}{4\pi\epsilon_0 R^4}$ (3) $\frac{Qr_1^2}{3\pi\epsilon_0 R^4}$ (4) 0

Ans. 2

Sol.

Consider a spherical shell having radius r and thickness dr

$$dq = \frac{Q}{\pi R^4} r \times 4\pi r^2 dr$$

$$q = \frac{4Q}{R^4} \int_0^{r_1} r^3 dr$$

$$q = \frac{Q \cdot r_1^4}{R^4}$$

Electric field at a distance r_1 from the center (in side)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{Q \cdot r_1^2}{R^4}$$

